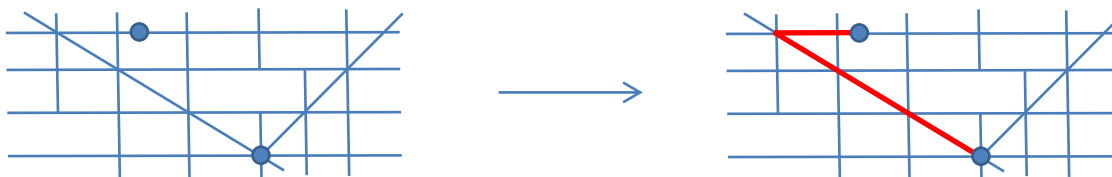


INDE 597  
 Homework 2  
 Due date 2/13

- 1) Several major delivery firms have eliminated left turns from their routes for various reasons [see, e.g., <https://www.cnn.com/2017/02/16/world/ups-trucks-no-left-turns/index.html>]. Your delivery firm wants to go a step further by having its trucks take as few turns as possible. Suppose that:
- The city your firm is in has  $n$  roads
  - Each road is a straight line segment
  - The set of all roads is topologically connected
  - Each road allows traffic in both directions
  - There are no forbidden turns at any intersection
  - Let  $a_{i,j}$  be 1 if roads  $i$  and  $j$  intersect and 0 otherwise
  - If  $x$  and  $y$  are two points on the same road,  $d_{x,y}$  is the distance between  $x$  and  $y$  along that road

Design a combinatorial algorithm which finds a path between two points in the road network of your city such that this path requires the smallest possible number of turns, and, subject to that, requires the shortest travel distance (see figure below for an example). Prove the correctness of your algorithm and determine its runtime complexity. You do not have to code up your algorithm for this homework.



Streets are pictured as blue segments. Among the paths between the marked points that require the fewest possible turns, the shortest such path is found (pictured in red).

- 2) You go to a party where some guests aren't friends, but for any partition of the guests into two nonempty groups, there is a person in the first group who is friends with someone in the second group.
- Suppose that each pair of guests at the party has exactly one mutual friend. Can there be an even number of people at the party?
  - Suppose that each pair of guests at the party has exactly two mutual friends. Can there be an even number of people at the party?

Justify your answers. This question may require you to search the literature.